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Transport implications of Fermi arcs in the pseudogap phase of the cuprates

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We derive the fermionic contribution to the longitudinal and Hall conductivities within a Kubo formalism, using a phenomenological Green's function which has been previously developed to describe photoemission data in the pseudogap phase of the cuprates. We find that the in-plane electrical and thermal conductivities are metalliclike, showing a universal limit behavior characteristic of a d-wave spectrum as the scattering rate goes to zero. In contrast, the c-axis resistivity and the Hall number are insulatinglike, being divergent in the same limit. The relation of these results to transport data in the pseudogap phase is discussed.

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The origin of the pseudogap phase is an on-going debate in the field of high-temperature cuprate superconductivity. Models of the pseudogap phase cover a broad range from competing density wave order all the way to preformed Cooper pairs. A principal characteristic of these theories is the nature of the electronic excitations. Although some photoemission studies are consistent with the presence of small pockets in the pseudogap phase,[1] others reveal that the Fermi surface breaks up into disconnected Fermi segments, known as Fermi arcs.[2] The length of these arcs scales linearly with $T/T^{*}(x)$, where $T^{*}(x)$ is the doping-dependent pseudogap temperature.[3] This behavior can be reproduced if one assumes a temperature-independent d-wave gap with a scattering rate proportional to T.[4,5]

If such temperature-dependent arcs do indeed exist, they should have profound implications for the nature of the transport in the pseudogap phase.[6] In particular, the transport properties of arcs should differ significantly from those expected for small pockets. To investigate this, we take a Green's function generated from a phenomenological self-energy that reproduces these arcs.[5,7] This was derived from a model of d-wave pairs without long-range order. We then construct the Kubo bubble and use this to calculate both the in-plane and c-axis longitudinal conductivities, as well as the Hall conductivity. We then connect our results to transport data for the cuprates.

The in-plane conductivity at $T=0$ is given by the Kubo formula[8]

$$\sigma_{xx} = \frac{2e^2\hbar}{\pi}\int \frac{d^3k}{(2\pi)^3} v_x^2(k)[\mathrm{Im}\, G(k,0)]^2, \qquad (1)$$

where v_x is the x component of the Fermi velocity,[9] $\mathrm{Im}\, G(k,0) \equiv \mathrm{Im}\, G^R(k,\omega=0)$ with G^R the retarded Green's function and the factor of 2 comes from summing over spin. Here, we ignore the anomalous FF contribution to the Kubo bubble that would generate the fluctuational contribution to the conductivity arising from the pairs themselves. That is, we treat only the density-of-states (fermionic) contributions and not the Aslamazov-Larkin and Maki-Thompson (bosonic) contributions,[10] leaving these for a later study.[11] As a consequence, we ignore vertex corrections since at lowest order this can only connect F lines and not G lines since the exchanged boson is a pair excitation.

Transforming the k integral in Eq. (1) to one over ϵ and ϕ, and ignoring the ϕ dependence of v and any c-axis dispersion, we have the planar conductivity

$$\sigma = \frac{e^2\hbar N v^2}{\pi d}\int d\epsilon \frac{d\phi}{2\pi}[\mathrm{Im}\, G(\epsilon,\phi,0)]^2, \qquad (2)$$

where N is the (two-dimensional) normal-state density of states (per spin), v the Fermi velocity, and d the c-axis thickness divided by the number of conducting planes.

We now consider a BCS model for the Green's function. In the so-called "single lifetime" version[4,5,7]

$$-\mathrm{Im}\, G(\epsilon,\phi,0) = \frac{\Gamma}{\epsilon^2+\Gamma^2+\Delta^2(\phi)}, \qquad (3)$$

where Δ is the pairing gap and Γ the inverse lifetime.[12] This form for G gives a good description of photoemission data in the pseudogap phase. In particular, if Γ scales as T, then the T dependence of the arc length is reproduced, as well as the variation in the spectral gap around the Fermi surface. Such scaling could be consistent with the T dependence of the inverse pair lifetime (which goes as $T-T_c$),[7] or vortex fluctuations above T_c (which roughly goes as T).[13] In general, linear T behavior is expected from the success of the marginal Fermi-liquid phenomenology in describing the cuprates.[14]

Substituting this form of G into the expression for σ, the ϵ integral is convergent, and setting its limits to infinity yields

$$\sigma = \frac{e^2\hbar N v^2\Gamma^2}{\pi d}\int_0^{\pi/2} \frac{dx}{(\Gamma^2+\Delta_0^2\cos^2 x)^{3/2}}, \qquad (4)$$

where in the d-wave case, $\Delta(\phi)=\Delta_0\cos x$ with $x=2\phi$. Performing the x integration

$$\sigma = \frac{e^2\hbar N v^2\lambda}{\pi d\Delta_0}E(\lambda), \qquad (5)$$

where $\lambda=\Delta_0/\sqrt{\Gamma^2+\Delta_0^2}$ and E is the complete elliptic integral of the second kind.

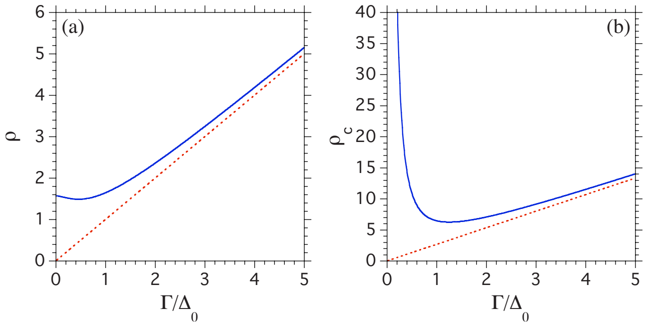

FIG. 1. (Color online) (a) planar and (b) c-axis resistivity versus Γ. The units for ρ are $2m^*/(e^2\hbar n)$ and for ρ_c are $\hbar^2 v^2 m^*/(d^2 e^2 \hbar n t_\perp^2)$, where $n/m^* = Nv^2/d$ with n the electron density and m^* the effective mass. The dashed lines are for $\Delta_0 = 0$.

This can be easily generalized[15] to the case of the c-axis conductivity by replacing $v^2/2$ by $t_\perp^2(\phi)d^2/\hbar^2$, where $t_\perp$ is the interlayer tunneling energy whose angle dependence goes as $\cos^2(2\phi)$.[16,17] This has a profound effect on the conductivity[18]

$$\sigma_c = \frac{2e^2 d N t_\perp^2 \Gamma^2}{\pi\hbar} \int_0^{\pi/2} \frac{dx \cos^4(x)}{(\Gamma^2 + \Delta_0^2 \cos^2 x)^{3/2}}. \tag{6}$$

Evaluating, one finds

$$\sigma_c = \frac{2e^2 d N t_\perp^2 \Gamma^2}{\pi\hbar \Delta_0^3 \lambda}[(2-\lambda^2)E(\lambda) - 2(1-\lambda^2)K(\lambda)], \tag{7}$$

where K is the complete elliptic integral of the first kind.

In Fig. 1, the inverse of σ (ρ) and σ_c (ρ_c) are plotted as a function of Γ. For zero Δ_0, they are proportional to Γ as expected. For nonzero Δ_0, it is seen that ρ saturates to a finite value in the limit that Γ goes to zero. This is the universal limit discussed by Lee.[19] The resistance has a weak minimum when $\Gamma/\Delta_0 \sim 0.46$ before increasing like Γ as in the normal state. The c-axis conductivity behaves quite differently because of the $\cos^4(2\phi)$ factor, which is peaked at $\phi = 0$, where the d-wave gap is maximal (thus killing off the universal behavior associated with the nodes). As a consequence, ρ_c diverges as $1/\Gamma^2$ [this behavior coming from the prefactor in Eq. (6)]. As Γ increases, a strong minimum is seen in ρ_c at $\Gamma/\Delta_0 \sim 1.26$ before increasing like Γ.

Although the above results are for $T=0$, we have done numerical studies which included the thermal factors in the Kubo bubble. Only minor differences were seen, and thus to a good approximation, the above $T=0$ formulas are sufficient. Therefore, if Δ_0 is constant, and Γ scales as T, as commonly assumed to describe the photoemission data, then the x axis of the above figures can be read as temperature.

We now turn to the Hall conductivity (current in the plane, field along the c axis) which is easily derived by insertion of a magnetic-field vertex into the Kubo bubble[20]

$$\sigma_H = -\frac{2e^2\hbar^2 N v^2 \omega_c}{3\pi d} \int d\epsilon \frac{d\phi}{2\pi} [\mathrm{Im}\, G(\epsilon,\phi,0)]^3, \tag{8}$$

where ω_c is the cyclotron frequency (eH/m^*c). Substituting G from Eq. (3) and performing the integral over ϵ

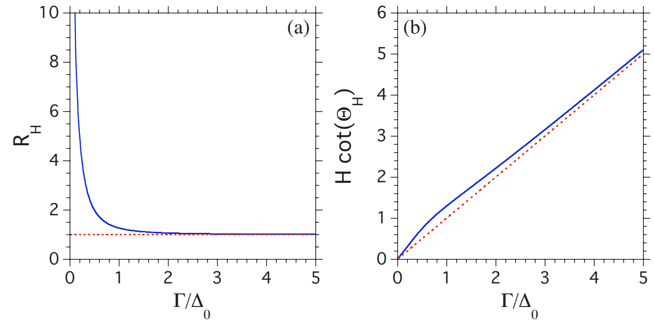

FIG. 2. (Color online) (a) Hall coefficient and (b) cotangent of the Hall angle versus Γ. The units for R_H are $1/(nec)$ and $H\cot(\Theta_H)$ are $2cm^*/(e\hbar)$. The dashed lines are for $\Delta_0 = 0$.

$$\sigma_H = \frac{e^2\hbar^2 N v^2 \Gamma^3 \omega_c}{2\pi d} \int_0^{\pi/2} \frac{dx}{(\Gamma^2 + \Delta_0^2 \cos^2 x)^{5/2}}. \tag{9}$$

Evaluating, one finds

$$\sigma_H = \frac{e^2\hbar^2 N v^2 \omega_c \lambda}{6\pi d \Gamma \Delta_0}[2(2-\lambda^2)E(\lambda) - (1-\lambda^2)K(\lambda)]. \tag{10}$$

Note that the Hall resistivity (ρ_H) is σ_H/σ^2, and the Hall coefficient (R_H) is ρ_H/H.

In Fig. 2(a), we plot the Hall number as a function of Γ, where it is seen to diverge as $1/\Gamma$ at small Γ. This behavior is due to the nodal contribution, thus generalizing the results of Ref. 19 to the Hall conductivity. In Fig. 2(b), we plot the cotangent of the Hall angle, $H\cot(\Theta_H) = \rho/R_H$, which vanishes as Γ goes to zero but increases as Γ for large Γ.

These results are easily generalized to their thermal counterparts.[21] The in-plane thermal conductivity is

$$\kappa_{xx} = \frac{2\pi\hbar k_B^2 T}{3} \int \frac{d^3k}{(2\pi)^3} v_x^2(k)[\mathrm{Im}\, G(k,0)]^2 \tag{11}$$

and thus we recover[22] the Wiedemann-Franz law $\kappa = \frac{\pi^2 k_B^2 T\sigma}{3e^2}$ (therefore, κ exhibits universal behavior as well). For the thermopower, S, and the Nernst, one must consider particle-hole asymmetry effects. In our simple model where Γ is taken as a momentum and frequency-independent constant, the only source for this is the density of states, N. As a consequence, we find that S/T is a constant $(\frac{\pi^2 k_B^2}{3e}\frac{d\ln N}{d\epsilon})$, and that the Nernst effect vanishes due to the Sondheimer cancellation (i.e., $\frac{\partial \tan(\Theta_H)}{\partial\epsilon} = 0$). These results will obviously change if a more sophisticated model is used for the self-energy.[21]

We now discuss implications of our results. The first point we wish to make is almost trivial. That is, transport data for various dopings scale as a function of $T/T^*(x)$.[23,24] Since Δ_0 scales with $T^*(x)$ (Ref. 25) and Γ with T,[14] then our transport results also scale as $T/T^*(x)$. Within our model, the scaling factor is such that $\Gamma = \sqrt{3}\Delta_0 T/T^*$, noting that $\Gamma/\Delta_0 > \sqrt{3}$ is the condition for gaplessness of $\mathrm{Im}\, G(\epsilon=0, \phi=0, \omega)$.[5] Since $\Delta_0/T^* \sim 2$, this reduces to $\Gamma \sim 2\sqrt{3}T$. The latter prefactor is close to π, a value consistent with the linear T part of the scattering rate extracted from transport data in the underdoped regime.[26]

Now, as the arc length scales with Γ, one might have naively expected that the planar resistivity would diverge as Γ goes to zero. This does not occur for the same reasons discussed by Lee.[19] That is, for a d-wave spectral gap, the residual conductivity is independent of the scattering rate. This result, though, does not hold for the c-axis resistivity, which indeed diverges. Experimentally, the in-plane resistivity is indeed metalliclike in the pseudogap phase whereas the c-axis resistivity is divergent.[27] So, this basic dichotomy of the cuprates is trivially explained by any model with a d-wavelike excitation spectrum.[28] On the other hand, the experimental in-plane resistance below T^* falls below that of the high-temperature linear T behavior of the normal state, whereas our model results fall above. This indicates that an extra bosonic contribution to the conductivity should exist, a likely source being the pairs themselves. In fact, it is well known that there is a significant contribution to the conductivity above T_c which follows the two-dimensional Aslamazov-Larkin form.[29] We will investigate these bosonic contributions in a future paper.[11] In regards to the c-axis resistance, the data are usually fit by an activated form,[24] rather than the power law we find. Our results, though, are obviously dependent on the precise form of $t_\perp(\phi)$ and $\Delta(\phi)$, and also to any temperature dependence of Δ_0 and $t_\perp$. Inclusion of impurity scattering will also cut off the divergence.

We now turn to the Hall conductivity. Our results are roughly consistent with the reported variation in R_H versus temperature in the pseudogap phase,[30–32] though our expression is more singular than the data. The simple function $\sqrt{a^2+b^2/\Gamma^2}$ does a good job of fitting the curve in Fig. 2(a). The above caveats about the temperature dependence of Δ_0, the inclusion of impurity scattering, and bosonic contributions to the conductivity should be kept in mind. In regards to Fig. 2(b), the actual Hall angle scales as T^2 rather than T as we find, indicating different lifetimes entering σ_H and σ as has been previously commented on.[33] In that context, we note that Γ in principle can be a function of angle, and that its temperature variation, as well as that of Δ_0, can also be angular dependent. Some evidence for this has been provided by photoemission,[34] tunneling,[35] and transport studies.[36] Finally, we note that a related study to ours was recently done by Smith and McKenzie,[37] where they considered other model Green's functions discussed in Ref. 5 as well.

In conclusion, we have calculated the temperature variation in various transport quantities within a simple model of a d-wave excitation spectrum with a linear T scattering rate, previously used to describe photoemission data in the pseudogap phase. We find that the in-plane electrical and thermal conductivities are metalliclike, but the c-axis and Hall conductivities are insulatinglike, in qualitative agreement with experimental transport data in the cuprates.

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